Swimming Pool

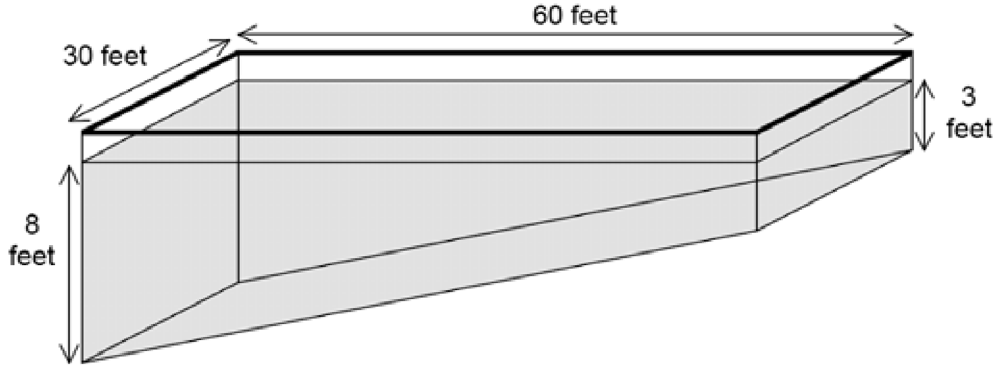
This problem gives you the chance to:
- work with trapezoids, rates and time graphs in a real context

This diagram shows a swimming pool.

The top of the swimming pool is a rectangle measuring 30 feet by 60 feet.

Two of the sides of the pool are trapezoids.

The water is 8 feet deep at the deep end and 3 feet deep at the shallow end.



1. Find the volume of water in the pool. ____________________ cubic feet
   Show your calculations.

The volume of water in the pool is 74,250 gallons.

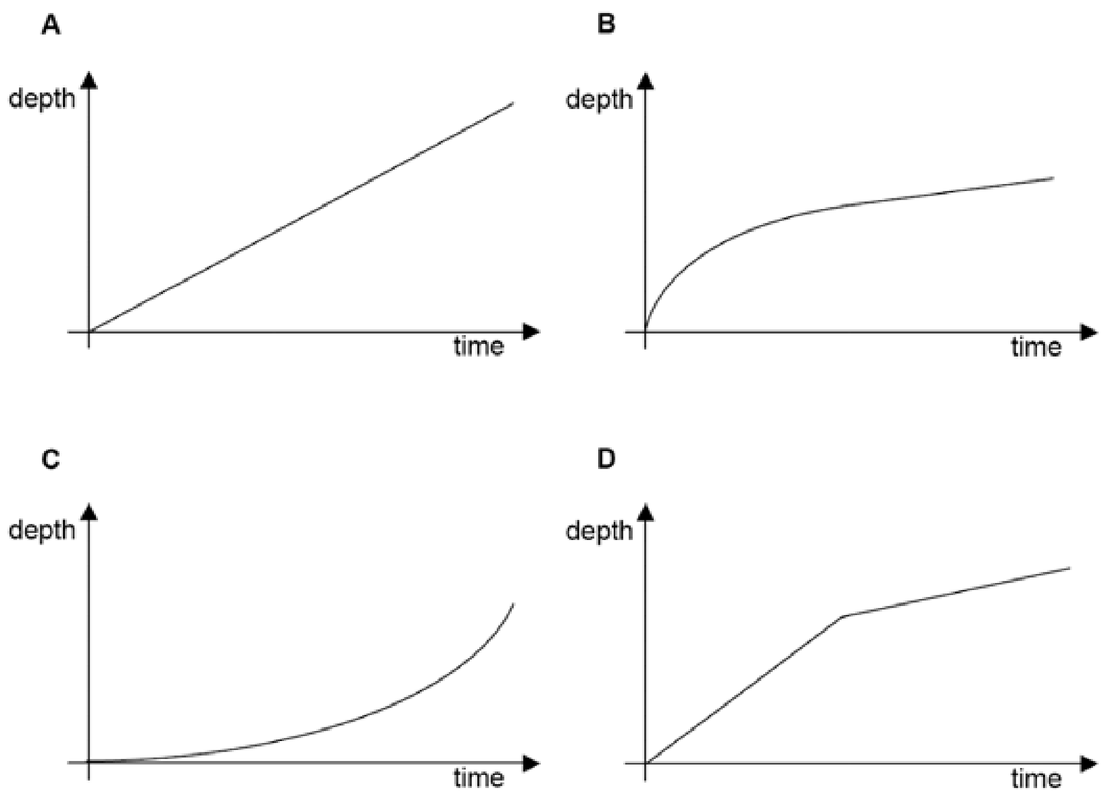
2. A pump fills the pool at a rate of 1 gallon per second. How long, in hours and minutes, does it take to deliver 74,250 gallons of water into the pool?

   __________ hours ___________minutes

   Show your calculations.

3. (a) Which of these graphs best represents the depth of the water in the pool as it is filled at a steady rate of one gallon per second?

Graph _______________



(b) Explain your reasons.

____

____

____

8

Swimming Pool — Rubric

The core elements of performance required by this task are: • work with trapezoids, rates and time graphs in a real context Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: **9,900** cubic feet	1	
Shows correct calculation such as: $60 \times \frac{(8+3)}{2} \times 30$	1	2
2. Gives correct answer: **20** hours, **37.5** minutes	1	
Shows correct calculation: dividing 74,250 by 60 x 60	1	
= 20.625 hours	1	3
3. (a) Gives correct answer: Graph **B**	1	
(b) Gives correct explanation such as: At first the depth increases quickly, but then more slowly as the water moves up the slope. For the final 3 feet, the depth increases at a constant rate.	2	
Partial credit A partially correct explanation.	(1)	3
Total Points		**8**

Algebra – Task 1: Swimming Pool

Work the task and examine the rubric.

What strategies might students use to find the volume of the swimming pool? See if you can find at least two.

How did you have to decompose the shape of the pool to use each strategy?

Now look at the work of you students for finding the volume of the pool. How many of your students put:

9900	43200	5400	74250	9000	1800	19800	14400	990	98	Other

What does the student know about the structure of the shape and volume for each of these wrong answers? What were students confused about?

How often do students in your class get the opportunity to break apart geometric figures?

How often do students in you class get the opportunity to justify where parts of a formula come from or to derive their own formula?

Do students get opportunities to solve problems that involve adding lines or pieces not present in the original diagram? How might this have helped students?

Did any of your students think about using the trapezoid as the base of the solid?

What is the mathematics involved in converting from seconds to hours? What confused students about changing the 20.625 hours into hours and minutes? Look at student work on the conversion. Can you sort their errors into categories?

Now look at student work on the graphs. How many of your students put:

B With good explanation	B With inc. explanation	A Steady rate	A Shape of pool	C	D

Did students make the connection that the deep end would fill quickly and then slow as the water moved up the slope of the pool?

Did students get confused about graphs, thinking that the shape of the graph should be related to the shape of the pool? Give some examples of this type of description.

Do you think some students didn't understand how the pool would fill? (Some students talked about filling in the shallow end first, then spilling over to the deep end or filling as if the top filled first then the bottom.)

Why do you think this task was so difficult for students?

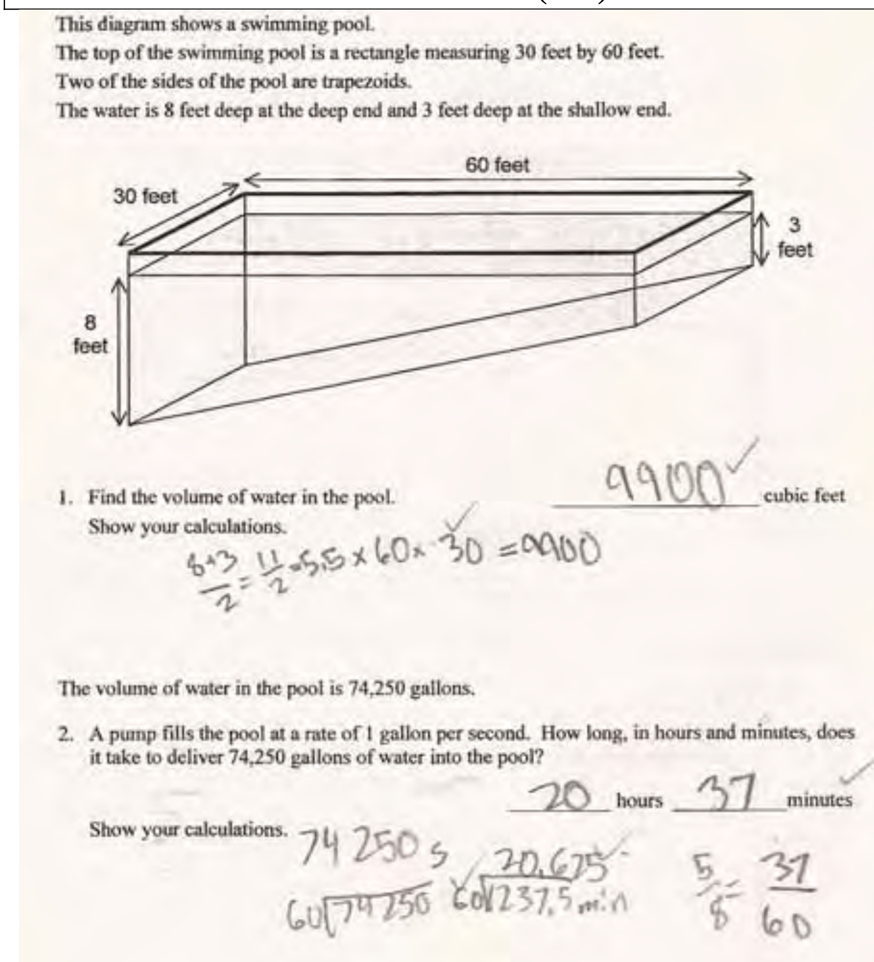
What types of experiences do students need with spatial visualization and composing/decomposing shapes?

What types of experiences do students need with understanding graphs?

Looking at Student Work on Swimming Pool:

There are three very different pieces of thinking needed to solve this task. For the first part, students need to combine or synthesize lots of learning about geometry, 3-dimensional shapes, and finding volume to derive a formula. For the second part, students need to do a conversion from seconds to hours and minutes. To complete this part of the task, students need to understand the relationship between a decimal quantity and a numerical value when decimal is parts of 60 instead of parts of 100. Again, students are being asked to synthesize previous knowledge to make sense of a less familiar application. The final part is reasoning about a time graph by looking at a geometric shape and thinking about how a pool would fill. For the first part of the task, students used four solution paths:

Solution Path 1: Some students understand that the volume of a prism is the area of the base times the height. To use this knowledge, the student needs to think of the sides of the pool, the trapezoids, as the base and the width of the pool (30) as the height. The area of the trapezoid would be (8+3)/2 times 60. So the total volume would be (8+3)/2 x 60 x 30. See the work of Student A.



This diagram shows a swimming pool.
The top of the swimming pool is a rectangle measuring 30 feet by 60 feet.
Two of the sides of the pool are trapezoids.
The water is 8 feet deep at the deep end and 3 feet deep at the shallow end.

1. Find the volume of water in the pool. 9900 cubic feet
   Show your calculations.
   $\frac{8+3}{2} = \frac{11}{2} = 5.5 \times 60 \times 30 = 9900$

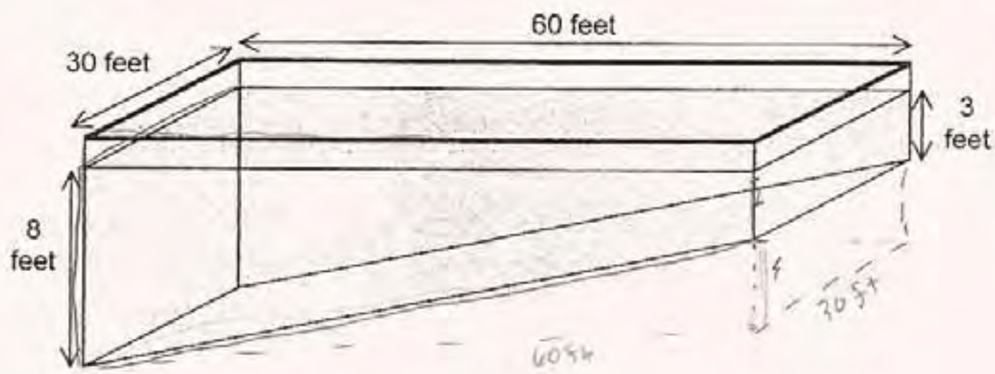
The volume of water in the pool is 74,250 gallons.

2. A pump fills the pool at a rate of 1 gallon per second. How long, in hours and minutes, does it take to deliver 74,250 gallons of water into the pool?
   20 hours 37 minutes
   Show your calculations. 74 250 s 60)74250 20.625 60)1237.5 min $\frac{5}{8} = \frac{37}{60}$

Notice that Student A is able to convert the decimal into minutes by changing the decimal to a fraction and then using a proportion.

Solution Path 2: This strategy involves adding extra lines or parts to the diagram to make more familiar shapes. First the student makes the shape into a rectangular prism with the dimensions 8ft. x 30 ft. x 60 ft., and finds the volume for this prism= 14,400 cu. ft. Then the student finds the volume of the triangular prism added to the original pool, (1/2 x 5 x 60)(30) = 4500 cu.ft. Finally the student subtracts the volume of the triangular prism from the volume of the rectangular prism. See the work of Student B.

Student B



1. Find the volume of water in the pool. Show your calculations.

   9900 ✓ cubic feet

   8 × 30 × 60 = 14400

   (60 × 5 ÷ 2)30 = 4500

   14400 − 4500 = 9900 ✓

The volume of water in the pool is 74,250 gallons.

2. A pump fills the pool at a rate of 1 gallon per second. How long, in hours and minutes, does it take to deliver 74,250 gallons of water into the pool?

   20 hours 37.5 minutes ✓

   Show your calculations.

   74,250 sec

   1,237.5 min

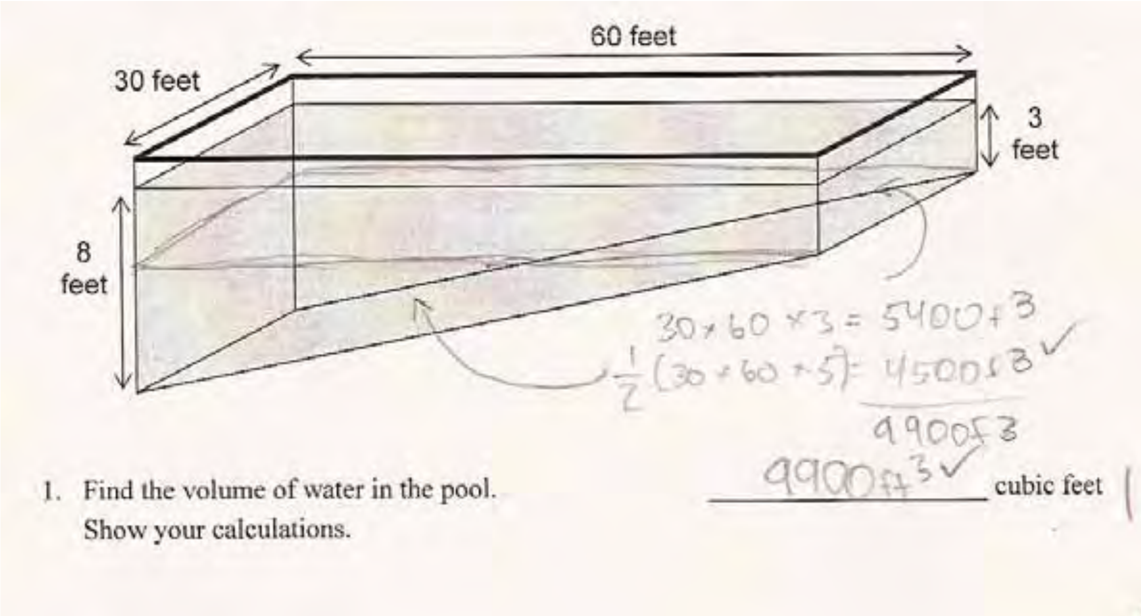
   74250.0 ÷ 60 = 1237.5 ✓

   1237.500 ÷ 60 = 20.625 ✓

Notice how Student B uses labels to keep track of what's known.

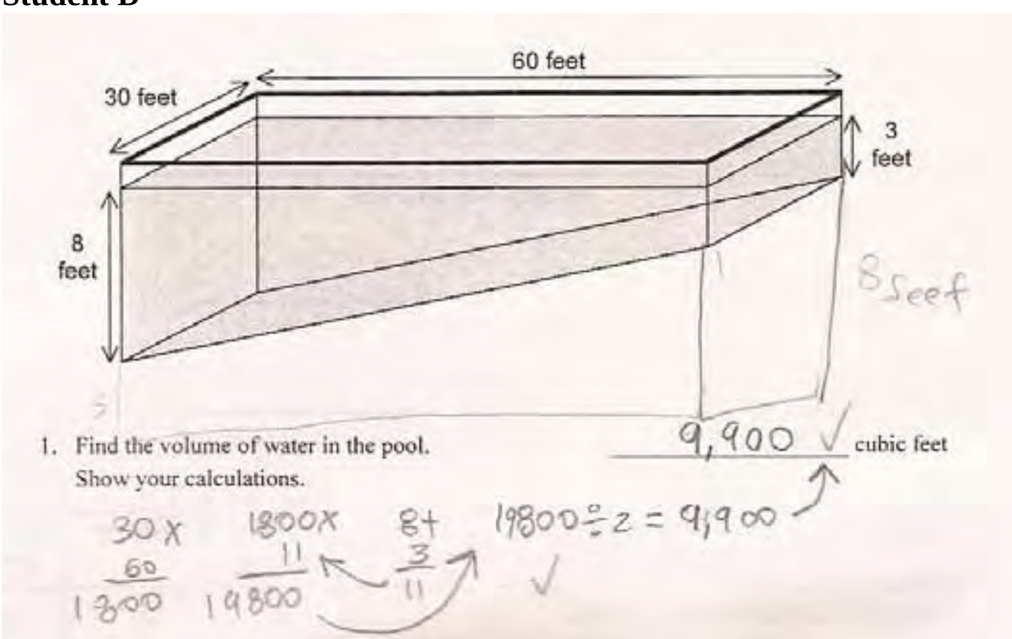
Solution Path 3: This involves decomposing the pool into a top part, a rectangular prism, and a lower part, a triangular prism, and add the volumes of the two parts together. Notice how Student C draws in the lines to show the two parts he is thinking about.

Student C



Solution Path 4: Another way to solve this problem is to duplicate the pool and fit the two pieces together to make a large rectangular prism with dimensions 11 x 30 x 60. This shape is easy to calculate the volume for because it fits the standard formula,
w x *h* x *l*. Then the original shape is just half as much. This method involves good spatial visualization.

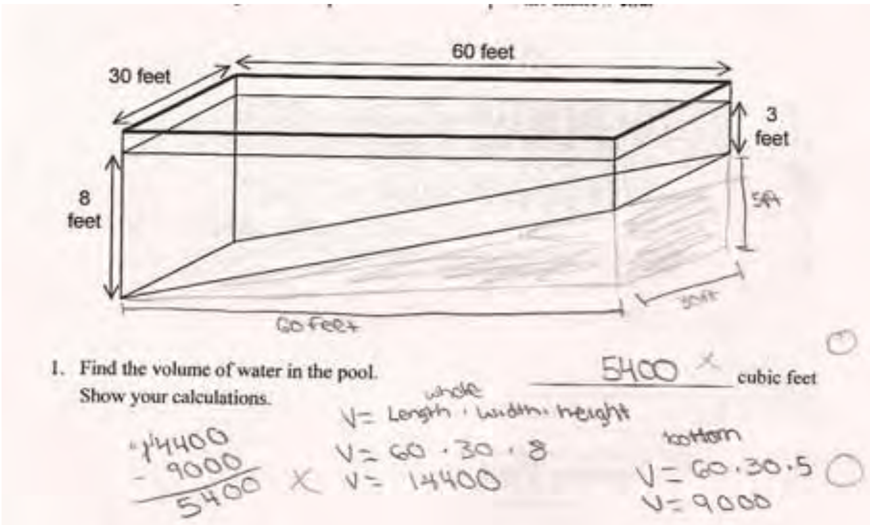
Student D



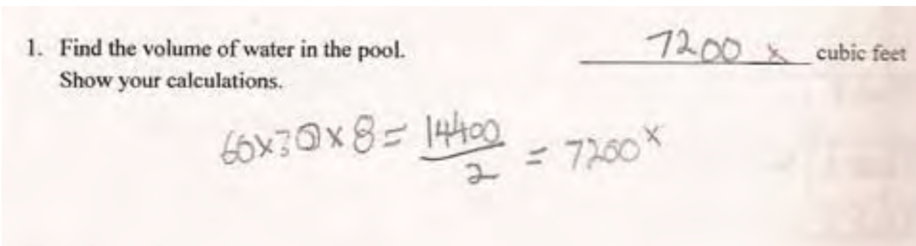
Algebra – 2006

Now, look at some student work below. Where does their thinking break down? What are they not understanding about composing/decomposing shape? What are they not understanding about volume?

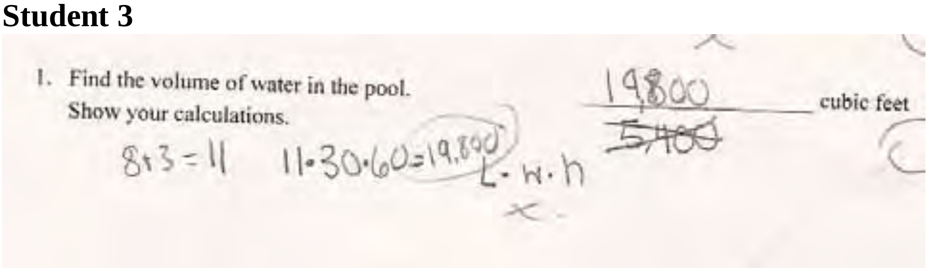
Student 1



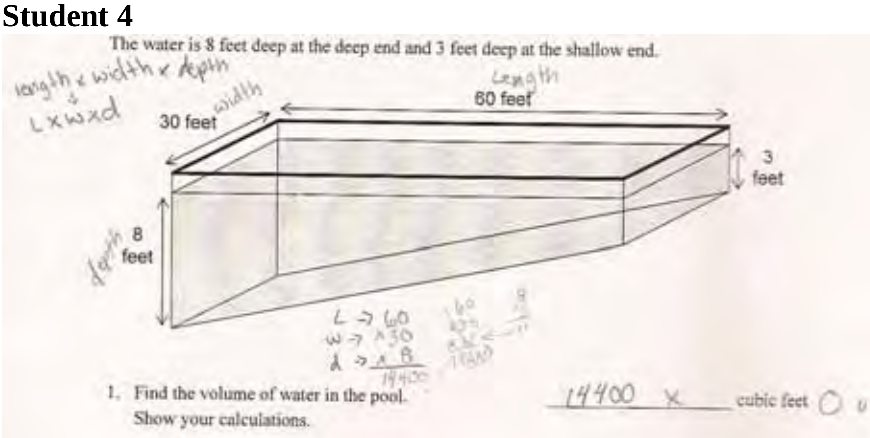
Student 2



Student 3

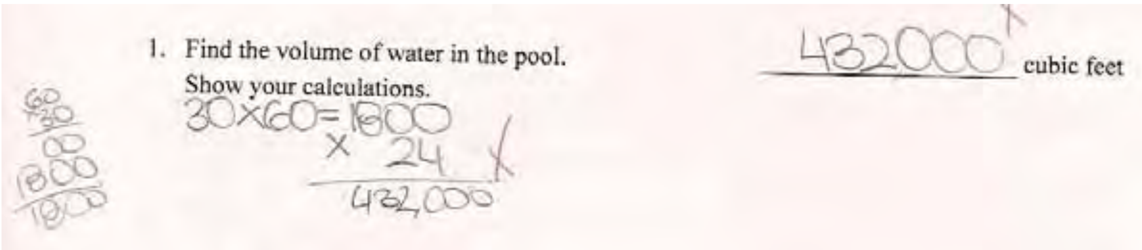


Student 4



Below are some examples of common error patterns by students in attempting to find volume. Student E has internalized the volume formula from l *x* w *x* h to multiply all the numbers in the figure. Student F combines dimensions and then multiplies. Student G finds the surface area of the pool, not the volume. Student H confuses the gallons with the volume in cubic feet.

Student E

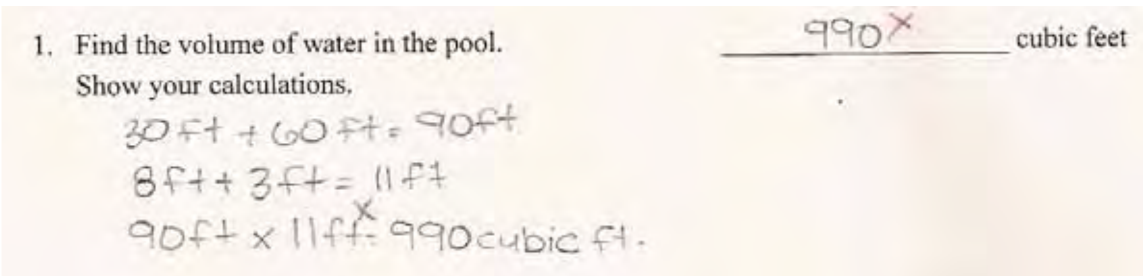


1. Find the volume of water in the pool.
Show your calculations.

30×60=1800
× 24
432,000

432000 cubic feet

Student F

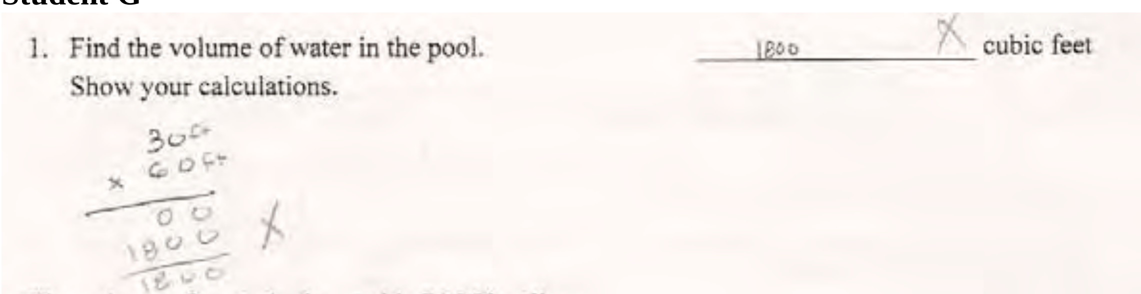


1. Find the volume of water in the pool.
Show your calculations.

30 ft + 60 ft = 90 ft
8 ft + 3 ft = 11 ft
90 ft × 11 ft = 990 cubic ft.

990 cubic feet

Student G

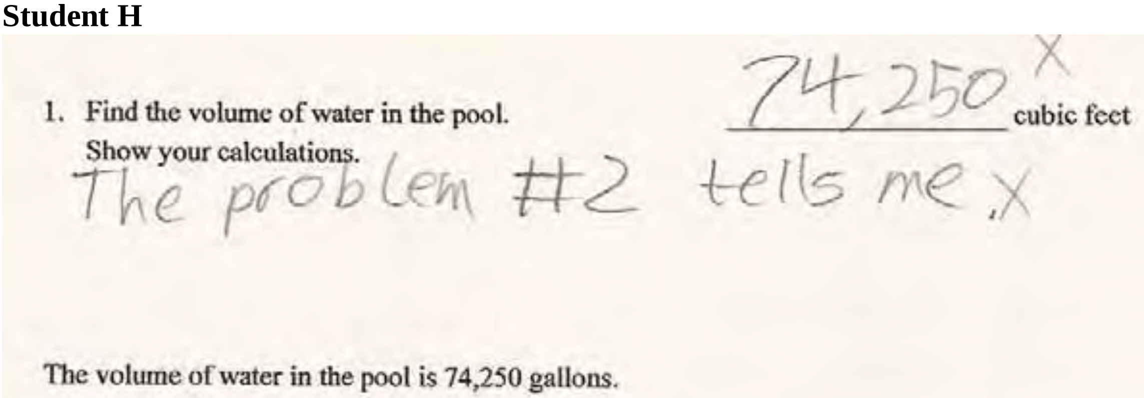


1. Find the volume of water in the pool.
Show your calculations.

30 ft
× 60 ft
00
1800
1800

1800 cubic feet

Student H



1. Find the volume of water in the pool.
Show your calculations.

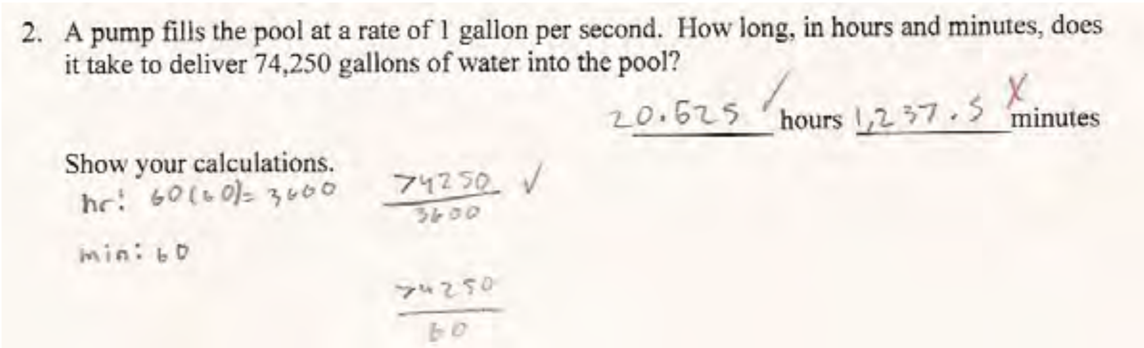
The problem #2 tells me.

74,250 cubic feet

The volume of water in the pool is 74,250 gallons.

Converting from seconds to hours proved challenging for students. Student I did not understand standard time notation and did two calculations; one for finding hours and one for finding minutes.

Student I


2. A pump fills the pool at a rate of 1 gallon per second. How long, in hours and minutes, does it take to deliver 74,250 gallons of water into the pool?

20.625 hours 1,237.5 minutes

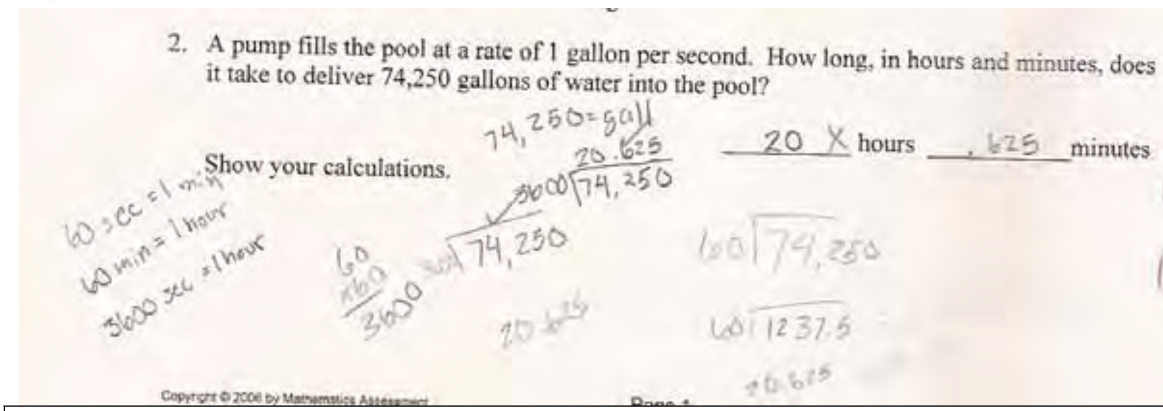
Show your calculations.

hr: 60(60) = 3600 74250/3600

min: 60 74250/60

Student J doesn't know what to do with the decimal part of the number and just moves it over to the minutes' side.

Student J

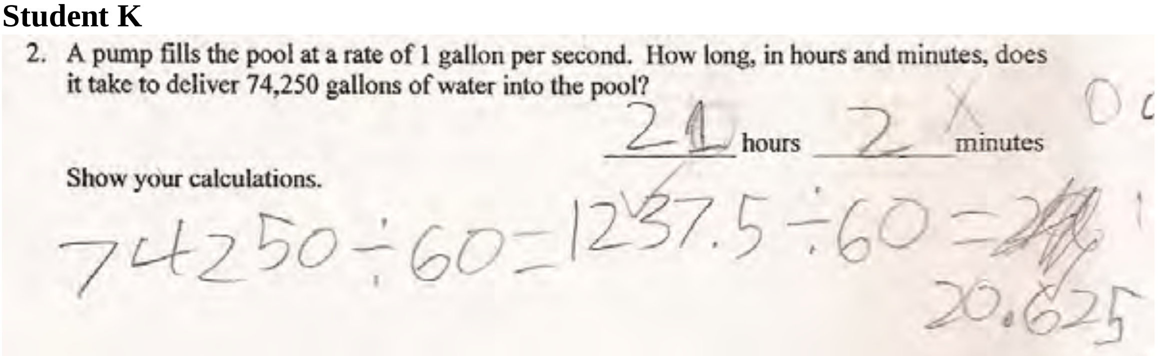

2. A pump fills the pool at a rate of 1 gallon per second. How long, in hours and minutes, does it take to deliver 74,250 gallons of water into the pool?

20 hours .625 minutes

Show your calculations.

Student K might have divided the decimal portion of the answer by 60 to get an additional hour using the remainder as the minutes. *What does this student not understand about the meaning of decimals?*

Student K


2. A pump fills the pool at a rate of 1 gallon per second. How long, in hours and minutes, does it take to deliver 74,250 gallons of water into the pool?

21 hours 2 minutes

Show your calculations.

74250 ÷ 60 = 1237.5 ÷ 60 = 20.625

Ideas about filling the pool and the graph:

- B – Since the bottom of the pool is sloped, the water will fill faster near the bottom, then slow as the water nears the 3 foot wall
- A- Because the slope of graph A is consistent showing a steady rate
- D- Represents the filling because of the way the tank was made. There is less space to fill in the bottom making it faster than filling the top

Other confusions about filling the pool and graph: *What might each student be thinking about?*

- C- because the pool is a trapezoid, so the water wouldn't be very deep when it started filling up, but it would be deeper as it reached the top.
- A- because its going straight up from 8 feet to 3 feet just like the swimming pool
- A – because it is a straight flat vertical surface
- C – because it shows the depth increasing little by little as the pool gets deeper and deeper
- C- As time goes on, the steady flow starts to fill up the deep end, making the pool deeper
- C- When the pool fills up, it will get more water where the hose is lying so one side will have more water since it's deeper

Algebra

Course One/Algebra Task 1 Swimming Pool

Student Task	Work with trapezoids, volume, rates and time graphs in the context of a swimming pool.
Core Idea 4 Geometry & Measurement	**Understand measurable attributes of objects; and understand the units, systems, and process of measurement.**
Core Idea 3 Alg. Properties & Representations	• Approximate and interpret rates of change, from graphic and numeric data.
Core Idea 1 Functions and Relations	• Analyze functions of one variable by investigating local and global behavior, including slopes as rates of change, intercepts and zeros.

Based on teacher observation, this is what algebra students knew and were able to do:

- Students were able to convert from seconds to hours, but were unsure what to do with the decimal

Areas of difficulty for algebra students:

- Finding volume of an unfamiliar shape
- Composing/ decomposing a shape into familiar parts
- Confusing a state rate of water flow with a steady rise in the depth of the pool
- Confusing the shape of the pool with the shape of the graph
- Not recognizing that after 5 feet the depth would increase at a steady rate

MARS Test Task 1 Frequency Distribution and Bar Graph, Course 1

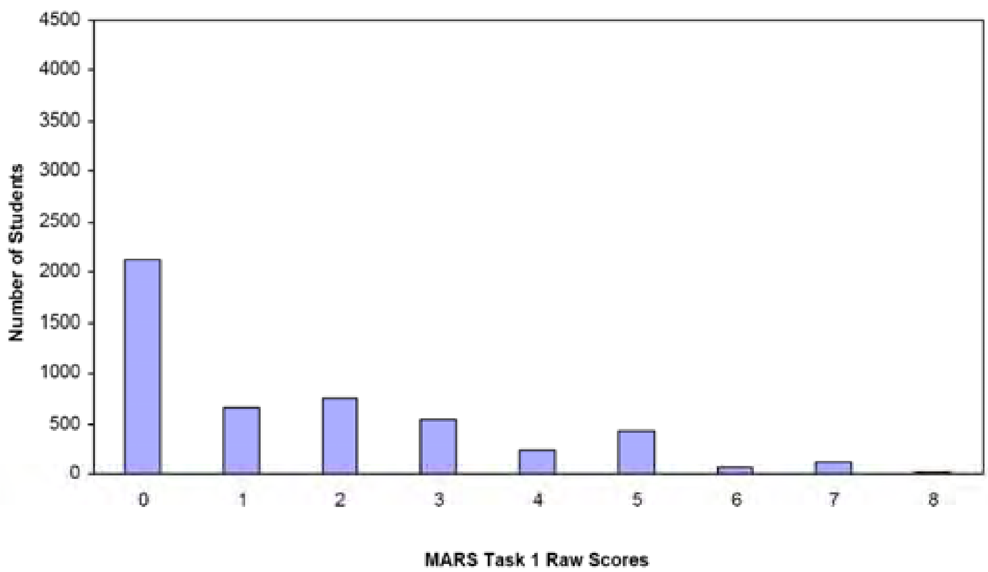
Task 1 - Swimming Pool

Mean: 1.67 StdDev: 1.95

Table 45: Frequency Distribution of MARS Test Task 1, Course 1

Task 1 Scores	Student Count	% at or below	% at or above
0	2130	43.1%	100.0%
1	655	56.3%	56.9%
2	747	71.4%	43.7%
3	538	82.3%	28.6%
4	240	87.1%	17.7%
5	428	95.8%	12.9%
6	70	97.2%	4.2%
7	114	99.5%	2.8%
8	25	100.0%	0.5%

Figure 54: Bar Graph of MARS Test Task 1 Raw Scores, Course 1



The maximum score available on this task is 8 points.
The minimum score for a level 3 response, meeting standards, is 5 points.

Less than half the students, 43.7%, knew to divide by 360, to change seconds to hours, and could do the calculation accurately. Some students, about 29%, could convert this answer to standard notation of hours and minutes by successfully changing the decimal from 0.625hrs. to 37.5 minutes. Only 13 % could make the conversion and then either find the volume of the pool or pick the correct graph with a partial explanation of why it was correct. Less than 1% of the students could meet all the demands of this task, including finding the volume of a trapezoidal prism and explaining how a graph of time and depth matches the situation of water filling the pool. 43% of the students scored no points on this task. 90% of the students with a score of zero attempted the task.

Swimming Pool

Points	Understandings	Misunderstandings
0	90% of the students with this score attempted the task.	Students did not understand how to convert seconds to hours. Some students only divided by 60. 11% did not attempt to do the conversion. Students had difficulty with decimal placement.
2	Students knew the process for conversions from seconds to hours (divide by 3600) and could calculate that accurately.	Students did not know what to do with the numbers after the decimal point or did not understand standard notation.
3	Students could successfully do all the steps of converting from seconds to hours and minutes.	Students had difficulty choosing the right graph. 60% of all students choose graph A. 18% picked graph C. Less than 5% picked graph D. Most students thought filling at a constant rate would be a straight line. Other students confused the shape of the pool for the shape of graph.
5	Students could convert from seconds to hours and minutes and either find the volume of the pool or pick the graph with a partial explanation of why it was correct.	10% of the students multiplied all 4 measures together to find the volume (43,200). 6% treated the pool as a rectangular prism with dimensions 8 x 30 x 60. 6% had an answer of 5400. Other popular answers were 7200, 9000, and 101.
8	Students could compose/decompose a 3-dimensional shape into familiar parts or add lines to make a familiar shape. This helped students to find volume of a trapezoidal prism. Students could convert from seconds to hours and minutes using standard notation. Students could reason about water filling a swimming pool and choose an appropriate time and depth graph, explaining how the shape of the graph matched the context.	

Implications for Instruction

Students need more experience with spatial visualization, including composing and decomposing geometric 2- and 3-dimensional shapes. By 7th and 8th grade students should start to work with taking slices of 3-dimensional shapes and being able to draw and measure those slices. Students need to think about rotations and flips of 3-dimensional shapes.

An important idea for solving geometric problems is the idea that lines can be added to 2-dimensional shapes or that extra shapes can be combined with 3-dimensional shapes. This ability to add on helps the problem solver find and use knowledge about more familiar shapes.

As students move into algebra, they should be pushed to generalize about geometric formulas; for example, moving from volume of a rectangular prism is "l *x* w *x* h" to thinking about volume as the area of the base times the height. This generalization can then apply to a wide variety of shapes, like cylinders and triangular prisms. A large piece of algebraic thinking is developing mathematical justification in words, diagrams, and symbols. Students should be able to connect the various representations. Students, at the algebra level, should also be encouraged to justify why formulas work; how do the various parts of the formula relate to the geometric context. Students should be able to make a strong case for why, when finding the area of a triangle, the length times width is divided by 2, or why when finding the area of a trapezoid the two bases are divided in two.

As students move through an algebra course they should have frequent experiences graphing functions of real-life contexts, such as time/distance graphs. Students need to see that graphs represent something different from the shape of object. For example, the graph of the height of a car on a ferris wheel over time is not a circle, but a peak shape, with an steady increase and decrease. Students need to discuss common misconceptions like this in order to see why these ideas are incorrect. Possible activities might include explaining the story of a graph, or given a story make a graph without the scale. Good examples can be found in the Language of Functions published by the Shell Centre.

Action Research - Developing Justification, Connecting Geometric and Symbolic Representations.

Have students work the MAC 4th Grade -Task2, 2004: Piles of Oranges or 5th grade 2001 Soup Cans. Can your students develop a rule for finding any number in the pattern? Try to get them to use a diagram to explain why the formula works and where the numbers come from.

Have students work the MAC Course1- 2000: Trapezoidal Numbers. See if students can find a pattern for finding the number of dots for any number in the pattern. See if they can use diagrams to explain why the formula works and where the numbers are represented in the diagram.

Performance Assessment Task
Swimming Pool Grade 9
The task challenges a student to demonstrate understanding of the concept of quantities. A student must understand the attributes of trapezoids, how to determine the area of trapezoids, how to determine the volume of a figure with two trapezoidal sides, and how to interpret rate and time graphs. A student must be able to approximate and interpret rates of change from graphic and numeric data. A student must make sense of this mathematics in a real-world context of the dimensions of a swimming pool.
Common Core State Standards Math - Content Standards
<u>**High School - Number and Quantity - Quantities**</u> **Reason quantitatively and use units to solve problems.** N-Q.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. N-Q.2 Define appropriate quantities for the purpose of descriptive modeling. N-Q.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
Common Core State Standards Math – Standards of Mathematical Practice
MP.2 Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. **MP.4 Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
Assessment Results
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including

the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.				
Grade Level	Year	Total Points	Core Points	% At Standard
9	2006	8	5	13 %